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LIGHT REFLECTIONS FROM SYSTEMS OF PLANE MIRRORS

TECHNICAL REPORT NO. ESD-TR-65-132

SEPTEMBER 1965

D. A. Berkowitz

Prepared for

DIRECTORATE OF COMPUTERS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 508G
Prepared by

THE MITRE CORPORATION
Bedford, Massachusetts
Contract AF19(628)-2390

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ABSTRACT

When light from an object is reflected successively from plane mirror surfaces, a rotated image is produced. An equation is derived and tabulated values presented for the amount of rotation in terms of directions of the incident and emergent light.

<u>REVIEW AND APPROVAL</u>

This technical report has been reviewed and is approved.

James W. Woodward Jr

JAMES W. WOODWARD, JR.
Major, USAF
Actg Chief, Computer Division
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SECTION I

INTRODUCTION

Triple-mirror corner reflectors, and their prism equivalent, are familiar optical devices. Their performance is measured by how much the incident beam and the reflected beams differ from being parallel. The deviation due to lack of mutual perpendicularity between the three mirror (or prism) faces has been studied, using the techniques of vector algebra[1, 2]* and an autocollimation technique described for measuring the deviation.[1]

The corner reflector is a special case of a larger class of multimirror systems. The requirement for other members of the class arises when there is a need, in an optical system, to have a rotated image (even number of mirrors) or a rotated mirror image (odd number of mirrors) of an object. A Dove prism[3] or its mirror equivalent[4] is a device which can be inserted into an in-line optical system to achieve the desired mirror image rotation. This report is concerned with mirror systems that produce rotated (mirror) images, in which the directions of incident and emergent light beams are not in-line nor otherwise parallel.

An earlier work in this field[5] provided a less than satisfactory formalism for selecting a set of mirror directions to produce a desired rotation, and failed to point out the simple relationship between object and image orientations in successive reflections. The formalism required the use of quaternions. Quaternions were concocted[6] by Sir William Hamilton. They are an algebraic system which can be used for dealing with three-dimensional problems in vector analysis.[7] They had previously been applied to triple-mirror systems.[8] The analysis which follows herein requires only conventional vector algebra and

*Numbers in brackets denote references listed at the end of this report.

trigonometry, and results in a single relationship between the various optical parameters that can readily be employed in the design of a mirror system.

SECTION II

ANALYSIS

The unit vectors $\underline{a}_0$, $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$, etc., describe the direction of an incident light beam, and its direction after each successive reflection (see Figure 1). For a Dove prism, $\underline{a}_0 = \underline{a}_3$ with three reflections; for a corner reflector, $\underline{a}_0 = -\underline{a}_3$ with three reflections (see Figure 2). The vectors

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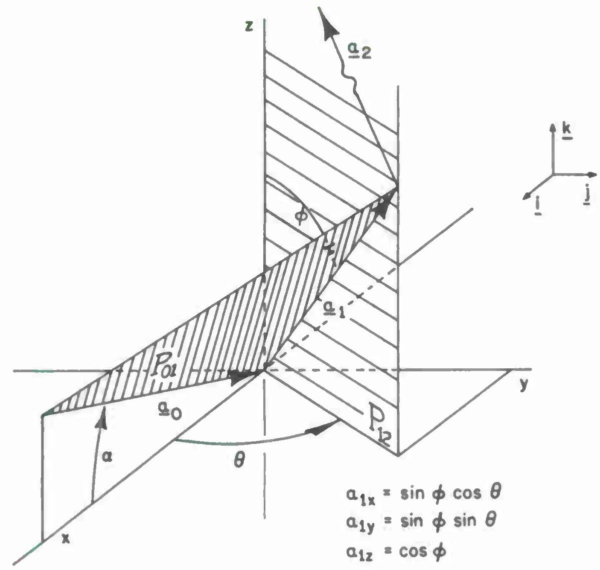

Figure 1. Vector Directions for Reflected Light Ray

$\underline{a}_0$ and $\underline{a}_1$ in Figure 1 determine a plane, P_{01}, which, in addition, contains the unit vector $\underline{n}_1$ perpendicular to the first mirror surface. If a vector in the object plane makes an angle, $90^o - \beta$, with plane P_{01}, then the mirror image of that vector (in the image plane) makes an angle, $90^o + \beta$, with plane P_{01}.

The vectors $\underline{a}_1$ and $\underline{a}_2$ in Figure 1 determine a plane P_{12} which, in addition, contains the unit vector $\underline{n}_2$ perpendicular to the second mirror surface. The vector $\underline{a}_1$ lies in the intersection of P_{01} and P_{12}. The angle

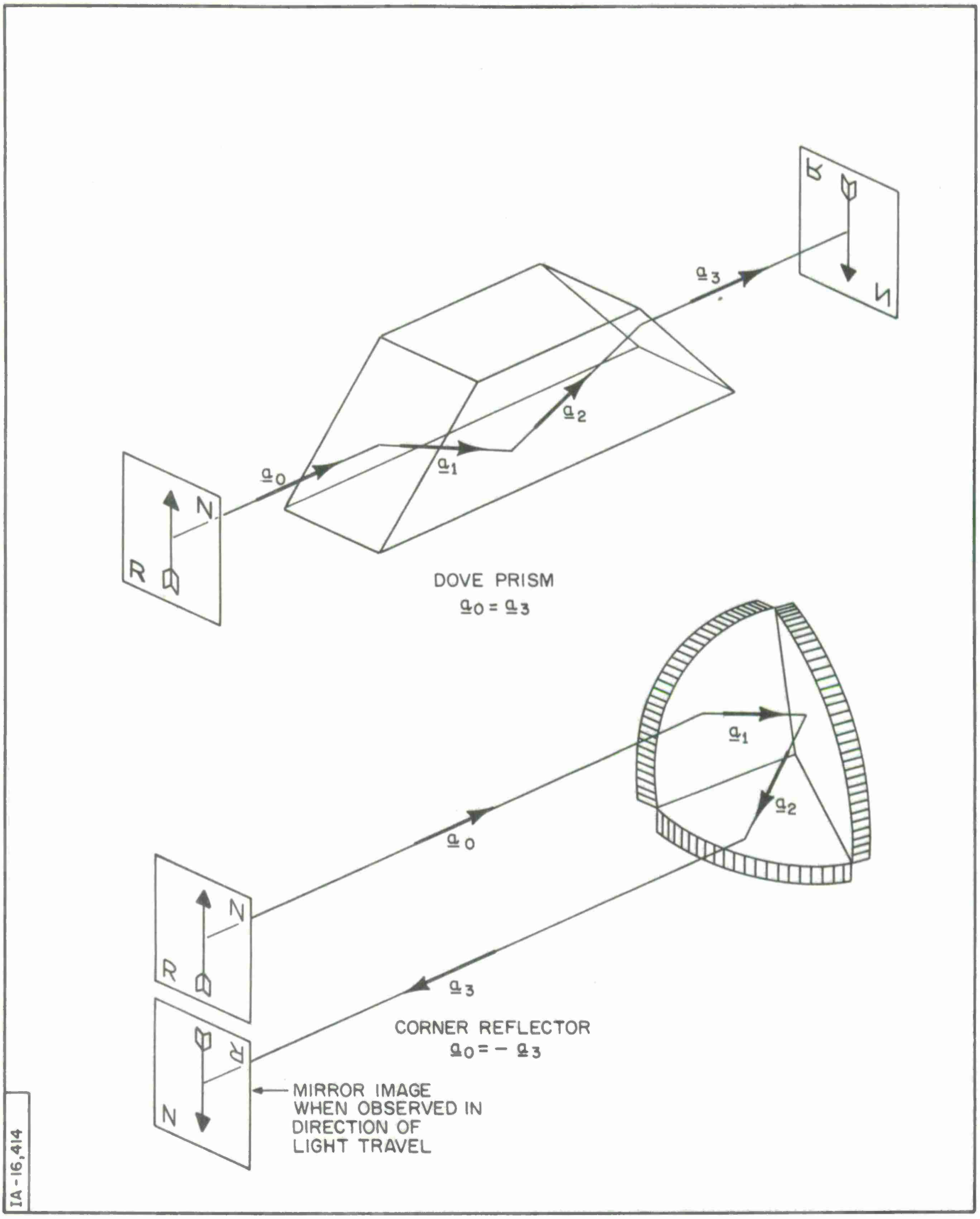

Figure 2. Vector Directions of Light Rays in a Dove Prism and a Corner Reflector

between P_{01} and P_{12} is A. Thus, the vector which makes an angle, $90^{\circ} + \beta$, with plane P_{01}, makes an angle, $90^{\circ} + \beta + A$, with plane P_{12}, and after reflection in mirror 2, makes an angle, $90^{\circ} - \beta - A$, with plane P_{12}.

Similarly, $\underline{a}_2$ and $\underline{a}_3$ determine plane P_{23} which makes an angle, B, with plane P_{12}. The vector which makes an angle, $90^{\circ} - \beta - A$, with P_{12} makes an angle, $90^{\circ} + \beta + A - B$, with plane P_{23} after reflection in mirror 3. With respect to P_{01}, the total rotation of the original object vector which has been triply reflected is $A - B + 2\beta$. Thus, in a Dove prism $(A = B = 0)$, if a vector makes an angle β with the plane containing the three normals to the mirror (or prism) faces, it will make an angle of 2β with its image.

The unit vectors are written in terms of their components according to the orientation of the coordinate system of Figure 1:

$$\underline{a}_0 = -\underline{i}\cos\alpha - \underline{k}\sin\alpha ,$$

$$\underline{a}_1 = a_{1x}\underline{i} + a_{1y}\underline{j} + a_{1z}\underline{k} , \tag{1}$$

$$\underline{a}_2 = -ca_{1x}\underline{i} - ca_{1y}\underline{j} + a_{2z}\underline{k} .$$

The vectors $\underline{a}_1$ and $\underline{a}_2$ lie in plane P_{12} which contains the z-axis; c is an arbitrary constant, and a_{2z} is arbitrary. Angle A between planes P_{01} and P_{12} is evaluated by computation of the dot-product of the normals to the planes.

$$\cos A = \frac{\left(\underline{a}_0 \times \underline{a}_1\right) \cdot \left(\underline{a}_1 \times \underline{a}_2\right)}{\left|\underline{a}_0 \times \underline{a}_1\right| \quad \left|\underline{a}_1 \times \underline{a}_2\right|} . \tag{2}$$

The quantities a_{2z} and c drop out of this expression, and, writing a_x, a_y, and a_z for a_{1x}, a_{1y}, and a_{1z}, Equation (2) takes the form

$$\cos A = \frac{\left(a_x^2 + a_y^2\right) \sin\alpha - a_x\, a_z \cos\alpha}{\left(a_x^2 + a_y^2\right)^{1/2} \left[1 - \left(a_x \cos\alpha + a_z \sin\alpha\right)^2\right]^{1/2}} \quad . \tag{3}$$

In terms of the angles θ and ϕ (see Figure 1),

$$\cos A = \frac{\sin\phi \sin\alpha - \cos\phi \cos\alpha \cos\phi}{\left[1 - (\cos\phi \sin\alpha + \sin\phi \cos\alpha \cos\theta)^2\right]^{1/2}} \quad . \tag{4}$$

Values of A corresponding to various values for α, θ, and ϕ are given in Table I. They were computed on an IBM 7030. (For negative values of α, read $180^{\circ} - \theta$ instead of θ, and take values listed for positive α from the table.

The direction cosines a_x, a_y, and a_z for the vector $\underline{a}_1$ can be calculated from the values of θ and ϕ taken from Table I. The orientation of the first mirror can be found by determination of the elements of the tensor which rotates $\underline{a}_0$ into $\underline{a}_1$.[1, 2] This tensor can be constructed simply. The vector diagram (Figure 3) in which $\underline{a}_0$ and $\underline{a}_1$ represent the incident and reflected rays, respectively, and $\underline{n}_1$ is the outward normal to the mirror, all vectors are of unit length, and $\underline{a}_1$ can be related to $\underline{a}_0$ and $\underline{n}_1$:

$$\underline{a}_1 = \underline{a}_0 - 2\left(\underline{n}_1 \cdot \underline{a}_0\right) \underline{n}_1 \quad . \tag{5}$$

If the tensor is to be written in dyadic form, and is the unit dyad, Equation (5) can be rearranged to the form,

$$\underline{a}_1 = \left(\quad \cdot \underline{a}_0\right) - 2\, \underline{n}_1 \left(\underline{n}_1 \cdot a_0\right), \tag{6}$$

and, finally,

$$a_1 = \left(\quad - 2\, \underline{n}_1 \underline{n}_1\right) \cdot \underline{a}_0 \quad . \tag{7}$$

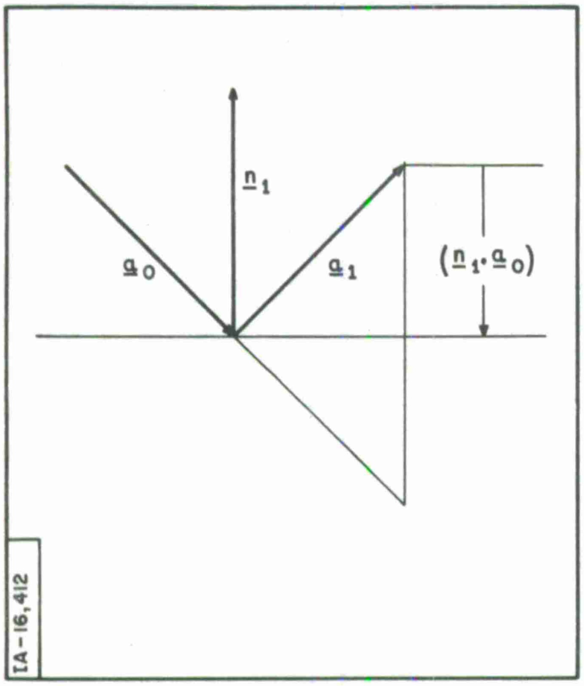

Figure 3. Vector Directions for Reflected Light Ray and Normal to Mirror Surface

The design procedure for a three-mirror, or equivalent prism, system should be to select $\underline{a}_0$ and $\underline{a}_3$ in the coordinate system of the optical instrument, most simply that system in which P_{12} is parallel to the z-axis. This determines values of α corresponding to $\underline{a}_0$ and $\underline{a}_3$. After selection of θ, values of ϕ for $\underline{a}_1$ and $\underline{a}_2$ must be found such that A and B (determined from the table) give the desired rotation A - B.

There is a sign ambiguity in the table; that is, there has been no statement regarding the sense of the angles of rotation. This is not because it is difficult to resolve, but rather it is the author's view that a simple laboratory mock-up of the mirror system is worth a page of diagrams and verbiage.

D. A. Berkowitz

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Table I

Values of A for Various Values of α , θ , and ϕ

α	θ	A					
		$\phi = 15$	$\phi = 30$	$\phi = 45$	$\phi = 60$	$\phi = 75$	$\phi = 90$
0	15	164.496	162.808	159.246	151.813	134.007	90.0
	30	149.133	146.310	140.769	130.893	114.147	90.0
	45	134.007	130.893	125.264	116.565	104.511	90.0
	60	119.147	116.565	112.208	106.102	98.499	90.0
	75	104.511	103.064	100.729	97.631	93.967	90.0
	90	90.000	90.000	90.000	90.000	90.000	90.0
	105	75.489	76.936	79.271	82.369	86.033	90.0
	120	60.853	63.435	67.792	73.898	81.501	90.0
	135	45.993	49.107	54.736	63.435	75.489	90.0
	150	30.868	33.690	39.232	49.107	65.854	90.0
	165	15.504	17.192	20.754	28.187	45.993	90.0
15	15	163.318	159.776	152.237	134.111	88.048	44.007
	30	146.906	140.935	130.225	111.896	86.033	61.813
	45	130.953	124.081	113.709	99.750	83.881	69.246
	60	115.529	109.050	100.729	91.187	81.501	72.808
	75	100.593	95.333	89.617	83.934	78.767	74.496
	90	86.033	82.369	79.271	76.936	75.489	75.000
	105	71.705	69.658	68.912	69.484	71.361	74.496
	120	57.472	56.787	57.911	60.899	65.854	72.808
	135	43.224	43.454	45.724	50.370	58.001	69.246
	150	28.897	29.494	31.946	36.936	45.993	61.813
	165	14.478	14.933	16.523	19.899	26.961	44.007
30	15	161.722	154.712	136.712	86.234	40.071	24.146
	30	143.956	132.696	112.208	82.369	56.294	40.893
	45	127.038	114.597	97.393	78.299	62.083	50.769
	60	111.063	99.462	86.386	73.898	63.687	56.310
	75	95.944	86.178	76.874	69.010	63.069	59.133
	90	81.501	73.898	67.792	63.435	60.853	60.000
	105	67.534	62.036	58.528	56.911	57.109	59.133
	120	53.862	50.194	48.663	49.107	51.572	56.310
	135	40.346	38.123	37.902	39.639	43.671	50.769
	150	26.891	25.705	26.100	28.187	32.600	40.893
	165	13.447	12.954	13.343	14.751	17.768	24.146

Table I (Concl'd)

Values of A for Various Values of α , θ , and ϕ

		A					
α	θ	$\phi = 15$	$\phi = 30$	$\phi = 45$	$\phi = 60$	$\phi = 75$	$\phi = 90$
45	15	158.999	142.436	84.682	34.045	19.876	14.511
	30	139.124	116.565	79.271	49.107	33.982	26.565
	45	120.859	99.030	73.675	54.068	42.088	35.264
	60	104.511	85.577	67.792	54.575	45.993	40.893
	75	89.477	74.061	61.517	52.671	47.057	44.007
	90	75.489	63.435	54.736	49.107	45.993	45.000
	105	62.221	53.142	47.339	44.138	43.081	44.007
	120	49.419	42.868	39.232	37.811	38.332	40.893
	135	36.898	32.443	30.361	30.105	31.610	35.264
	150	24.536	21.801	20.754	21.052	22.789	26.565
	165	12.252	10.960	10.547	10.861	12.017	14.511
60	15	151.900	83.496	25.537	14.278	10.308	8.499
	30	127.828	76.936	39.232	25.108	19.039	16.102
	45	108.364	70.266	44.294	31.666	25.387	22.208
	60	92.293	63.435	44.830	34.715	29.294	26.565
	75	78.399	56.395	42.838	35.174	31.024	29.147
	90	65.854	49.107	39.232	33.690	30.868	30.000
	105	54.136	41.542	34.456	30.660	29.036	29.147
	120	42.922	33.690	28.754	26.330	25.663	26.565
	135	32.007	25.561	22.293	20.881	20.856	22.208
	150	21.264	17.192	15.225	14.503	14.765	16.102
	165	10.613	8.644	7.726	7.436	7.665	8.499
75	15	82.753	14.112	7.538	5.378	4.112	3.967
	30	75.489	24.133	13.859	10.128	8.413	7.631
	45	68.194	29.425	18.293	13.802	11.676	10.729
	60	60.853	31.146	20.754	16.194	13.992	13.064
	75	53.455	30.469	21.470	17.293	15.271	14.511
	90	45.993	28.187	20.754	17.192	15.504	15.000
	105	38.464	24.803	18.895	16.032	14.738	14.511
	120	30.868	20.641	16.140	13.967	13.057	13.064
	135	23.211	15.924	12.695	11.156	10.574	10.729
	150	15.504	10.820	8.743	7.769	7.440	7.631
	165	7.761	5.471	4.455	3.985	3.841	3.967

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